M. Sc. Part – I (D.E.) – 2011
Assignment

Paper – I

Unit-I

1. Let G be a group of order 665.
   i) Show that G has a unique subgroup of order 5.
   ii) Show that G is cyclic.

2 ½ + 2 ½

Unit-II

1. i) Define integral domain.
   ii)Prove that the characteristic of an integral domain is either zero or a prime. 1 + 4

Paper – II

Unit-I

1. What do you mean by Rienanon – Stieltjes integration? Let α be a monotonically increasing function over [a,b]. If f is continuous on [a,b], then prove $f \in R(\alpha)$ on [a,b]. 2 + 3

Unit-II

1. When a function f $IR^m \longrightarrow IR^n$ is said to be differentiable?
   Show that for the function f: IR2 $\longrightarrow$ IR defined by

$$f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2}, (x,y) \neq (0,0), \\ 0, \text{otherwise} \end{cases}$$

   is not differentiable at the origin, although all the partial derivatives exist at that point.

1+ 4

Paper - III

(Topology – I)

Unit-I

1. Show that the function f: IR_ℓ -> IR defined by $f(x) = [x^2]$ = the greatest integer not greater than x^2 is continuous. What will happen if we replace lower limit topology by the usual one on the domain? 2 ½
2. Show that every metric space determines a unique topological space. Does the converse true? Justify your answer. Show that the metric spaces (IR^2, d_e) and (IR^2, d_s) determine same topology on IR^2. 2 ½

(Topology-II)

Unit-II

1. Examine whether the following sets are connected or not. 5 x 1
   a) IR with lower limit topology.
   b) IR with cofinite topology.

c) IR with co-countable topology.
d) IR^2 – C, with the subspace topology induced by the product topology on IR^2 where C is a countable subset of IR^2.
e) IR^2 – B, with the subspace topology induced by the product topology on IR^2 where B is a bounded subset of IR^2.

Paper – IV

Unit-I

1. Find the bilinear transformation which maps the points $z = \infty, i, 0$ into points $w = 0, i, \infty$ respectively. 5

Unit-II

1. Prove that Log M(r) is an increasing convex function of logr. 5

Paper – V

Unit-I

1. What do you mean by a differentiable manifold? Prove that the real projective space IRP^n is a differentiable manifold. 2 + 3

Unit-II

1. Define a linear connection. Also define the Torsion tensor and the curvature tensor of a linear connection. 3 + 1 + 1