Directorate of Distance Education
North Bengal University

PG Part II (Session: 2010 – 11) Assignment - I Subject: Mathematics

F.M. 100

Paper – VI, Unit – I: **5+5=10**

1. Given a signed measure space (X, s, λ). Let f be an extended real-valued s - measurable function on X . If f is λ semi-integrable on X, show that,

$$\left| \int_X f \, d\lambda \right| \leq \int_X |f| \, d|\lambda|.$$

2. Let S and T be algebras of subsets of sets X and Y respectively. Let $\{A_i \times B_i \; ; \; i = 1,2,...n\}$ be a collection in the semi-algebra S x T of subsets of X x Y and let $E = \bigcup_{i=1}^{n} (A_i \times B_i)$. Prove that $E = \bigcup_{j=1}^{p} (A_i^1 \times B_j^1)$ where $\{A_j^1; \; j = 1,2,.....p\}$ is a collection in S and $\{B_j ; \; j = 1,2,.....p\}$ is a collection in T .

Paper – VI, Unit – II: **6+4=10**

1. Let (X, ρ) be a complete metric space and let T be an operator mapping X into itself such that $\rho(T_x, T_y) \leq \alpha \, \rho(x, y)$ for all $x, y \in X$ where $0 < \alpha < 1$. Show that there exists a unique fixed point of the operator T in X.
2. Prove that on a finite dimensional linear space any norm $\| \; \|_1$ is equivalent to any norm $\| \; \|_2$.

Paper – VII, Unit – I: **8+2=10**

1. Describe fully Jacobi's method of solving first order Partial Differential equation and solve by this method the equation $x^2 p^2 + y^2 q^2 = 4$.
2. Show that the Neumann problem is stable.

Paper – VII, Unit – II: **8+2=10**

1. Derive Euler-Lagrange equation and using this equation find the equation of the curve, which when revolves about its axis, generates a surface of revolution of minimum area.
2. Show that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$, where H is the Hamiltonian function.

Paper – VIII, Unit – I: **10**

1. Derive the general motion for irrotational flow of a fluid element.

Paper – VIII, Unit – II: **10**

1. Show that the Bernoullis equation for compressible fluid can be obtained by the formula $\frac{2C}{\gamma - 1} dc + q\, dq = 0$

Paper – IX, Unit – I: **6+4=10**

1. What are ordinary derivative and covariant derivative? Why it is required to define the two kinds of derivative stated above? Show that $A_{\mu;\nu;k} - A_{\mu;k;\nu} = R^{\lambda}_{\mu\nu k} A_{\lambda}$, where $R^{\lambda}_{\mu\nu k}$ is Riemann Tensor.
2. Using the Riemann tensor obtain Bianchi Identity and hence show that covariant divergence of Einstein tensor vanishes.

Paper – IX, Unit – II: **10**

1. State the cosmological principles? Obtain a suitable line element which permits such principle. Derive the dynamical equations of cosmology corresponding to the above line element. Hence obtain Raychaudhuri equation.

Paper – X, Unit – I: **6+4=10**

1. Explain the phenomenon 'Interlocking of lines' with examples.
2. Write a note on limb-darkening.

Paper – X, Unit – II: **10**

1. Write down Milne's first integral equation and solve it by Ambertzumian's method.

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PG Part II (Session: 2010–11) Assignment - II Subject: Mathematics

F.M. 100

Paper – VI, Unit – I: **5+5=10**

1. Let g be a real-valued increasing function on $\Re$ and l_g be the set function on the semi-algebra $\Im_{oc}$ and μ_g^* be the outer measure on $\Re$ based on l_g Show that
   (i) l_g is a countably additive set function with $l_g(\phi) = 0.$
   (ii) μ_g^* is a regular outer measure on $\Re$.
2. Prove that the monotone class generated by an algebra S of subsets of a set X is equal to the $\sigma-$algebra generated by S.

Paper – VI, Unit – II: **10**

1. Show that in a metric space (X, d) the following conditions are equivalent :
   (i) X is compact.
   (ii) Every sequence in X has a convergent subsequence.
   (iii) X is complete and totally bounded.

Paper – VII, Unit – I: **5+5=10**

1. Solve the Dirichlet Interior problem for a rectangle of sides a and b.
2. A homogeneous rod of conducting material of length $100\,cm$ has its ends kept at zero temperature and the temperature initially is $u(x, 0) = \begin{cases} x, & 0 \le x \le 50 \\ 100 - x, & 50 \le x \le 100 \end{cases}$.
   Find the temperature at any time.

Paper – VII, Unit – II: **5+5=10**

1. Derive Hamilton-Jacobi equation and show that when solving this equation we are at the same time obtaining a solution of the mechanical problem.
2. Triangle ABC is a normal cross-section of a uniform triangular prism of infinite length and of mass M per unit length. Show that the attraction at the point A is the resultant of $(4GM\angle A)/a$ perpendicular to BC and $(4GM/a)\log(b/c)$, parallel to BC.

Paper – VIII, Unit – I: **10**

1. What does the complex function $W = \dfrac{z+5}{(z+1)(z+3)}$ represent in the fluid mechanics? Find ϕ and ψ and interprete them.

Paper – VIII, Unit – II: **10**

1. Show that velocity of sound (C) can be obtained by the formula $C^2 = \left(\frac{\partial p}{\partial \rho}\right)_0 = \gamma \frac{p_0}{\rho_0}$ also show that $C \, \alpha \, \sqrt{T_0}$. Symbols have their own meaning.

Paper – IX, Unit – I: **10**

1. What are the experimental tests of General theory of relativity? Describe in details one of them.

Paper – IX, Unit – II: **10**

1. What do you mean by particle horizon and event horizon? State its physical importance.
2. What are the different cosmological models? Describe in details the steady state theory of the universe.

Paper – X, Unit – I: **10**

1. Deduce the equation of transfer for non-coherent scattering.

Paper – X, Unit – II: **10**

1. Deduce the equation of transfer in a semi-infinite, isotropically scattering atmosphere and solve it by Chandrasekhar's Discrete Ordinate Method.