



# DIRECTORATE of DISTANCE EDUCATION

## North Bengal University

PG Part II (2015-16)

Assignment

Subject: Mathematics

F.M. 100

### Paper – VI, Unit – II:

5+5=10

1. If  $E$  is a measurable set such that  $0 < \mu(E) < \alpha$ . Then show that there is a positive set  $A$  contained in  $E$  with  $\mu(A) > 0$ .
2. Let  $\{E_n\}$  a sequence of disjoint measurable sets and for each  $n$ , let  $f_n$  be a function in

$L^p$  ( $1 \leq p \leq \alpha$ ) which vanishes outside  $E_n$ . Let us set  $f = \sum_{n=1}^{\alpha} f_n$ . Then show that  $f \in L^p$  if

$$\sum_{n=1}^{\alpha} \|f_n\|^p < \alpha.$$

### Paper – VI, Unit – II:

5+5=10

1. Let  $f$  be a functional on  $l_1^\alpha$  defined by  $f(x_1, x_2, x_3) = ax_1 + bx_2 + cx_3$ . Find  $\|f\|$ . Also find  $\|f\|$ , when  $f$  is considered as a functional on  $l_2^3$  and  $l_\alpha^3$ .
2. Show that any orthonormal sequence in a Hilbert space converges weakly to zero.

### Paper – VII, Unit – I:

5+5=10

1. If  $u$  and  $v$  are harmonic, show that  $uv$  is also harmonic iff  $\nabla u$  and  $\nabla v$  are orthogonal.
2. Find the integral surface of  $z(xp + yq) = x^2 + y^2$  through the parabola  $x = 2, z = y^2$ . Find general integral of  $yzp + xzq = x + y$ .

### Paper – VII, Unit – II:

5+5=10

1. Deduce Language equation of motion using D'Alembert's principle for holonomic system. Deduce Hamilton's principle using D'Alembert's principle.
2. Establish the relation between Poisson bracket and Lagrange bracket. Show that both Poisson bracket and Lagrange bracket are invariant under canonical transformation.

### Paper – VIII, Unit – I:

5+5=10

1. Show that uniform stream of velocity  $U$  can be obtained as the limit  $a \rightarrow \alpha$ , of the field due to a source of strength  $2\pi\alpha^2 U$  at  $(-a, 0, 0)$  and a sink of strength  $-2\pi\alpha^2 U$  at  $(a, 0, 0)$ .

2. Verify that  $\psi = \left( \frac{A}{r^2} \cos \theta + Br^2 \right) \sin^2 \theta$  is a possible form of Stokes' stream function; find the corresponding velocity potential.

**Paper – VIII, Unit – II:**

**5+5=10**

1. By introducing  $\psi$  in the Navier-Stokes equation, show that the order of the equation can be increased by 2.
2. Define displacement thickness, momentum thickness and energy. Obtain a relation among them.

**Paper – IX, Unit – I:**

**5+5=10**

1. Determine the equation for geodesics on the 2 dimensional space defined by

$$ds^2 = (dx^1)^2 + \left[ (x^1)^2 + \lambda^2 \right] (dx^2)^2. \quad \lambda \text{ being a constant.}$$

2. Determine the gravitation red shift formula for the Schwarzschild space-time.

**Paper – IX, Unit – II:**

**5+5=10**

1. Derive the Robertson-Walker line element in a homogeneous and isotropic space-time.
2. Determine the Hubble constant in the
  - i) Radiation dominated and
  - ii) Matter dominated era of the Einstein-de-Sitter model.

**Paper – X, Unit – I:**

**5+5=10**

1. Solve the equation of transfer by Edington method.
2. Solve the equation of transfer for coherent scattering when  $\eta$  is a function of  $\zeta$ .

**Paper – X, Unit – II:**

**5+5=10**

1. Establish the relation  $H(u)H(-u) = T(M) - 6$ .
2. Show that the expression for  $B(\zeta)$  is exact.