M.A., M.Sc. Part - II (Annual) Examination, 2013
(Under DE Stream)

MATHEMATICS

Paper - VI

Unit - I

Time - Two Hours Full Marks - 40

Answer *one* from **Section - A**, *three* from **Section - B** and *all* of **Section - C**.

The figures in the margin indicate full marks.

Section - A

1. (a) Let (X, S, λ) be a signed measure space. Prove that for every Jordan decomposition of λ, a Hahn decomposition of (X, S, λ) exists and conversely. 9

(b) Let λ be a σ-finite signed measure and μ be a positive measure on a measurable space (X, S). Show that two signed measures λ_1 and λ_2 exists uniquely on S such that $\lambda_1 << \mu$, $\lambda_2 \perp \mu$ and $\lambda = \lambda_1 + \lambda_2$. 9

2. (a) Consider the product measure space $(X \times Y, \sigma(S \times T), \mu \times v)$ of two σ-finite measure (X, S, μ) and (Y, T, v). If W is the collection of all those sets $E \in \sigma(S \times T)$ for which the function $\mu(E^y) : Y \to \overline{\mathbb{R}}$ is T-measurable and

$$(\mu \times v)(E) = \int_Y \mu(E^y)\, dv.$$

Prove that $\sigma(S \times T) \subset W$. 9

(b) If $(\mathbb{R}, M(\mu_g^*), \mu_g)$, $(\mathbb{R}, M(\mu_h^*), \mu_h)$ and $(\mathbb{R}, M(\mu_{g+h}^*), \mu_{g+h})$ are three Lebesgue-Stieltjes measure spaces determined by real-valued increasing functions g, h and $g+h$ on $\mathbb{R}$, show that

$$\mu_g << \mu_{g+h} \text{ and } \mu_h << \mu_{g+h} \text{ on } B_{\mathbb{R}}.$$ 9

Section - B

3. Let μ be a finite measure and $\{v_n\}$ be a sequence of finite measures on a measurable space (X, S) such that $v_n << \mu$ for each n. Assume that $\lim v_n(E)$ exists in $\overline{\mathbb{R}}$ for each $E \in S$. Show that the set function $v : S \to [0, \infty]$ defined by $v(E) = \lim v_n(E)$ for each $E \in S$, is a measure such that $v << \mu$. 6

4. Let μ be a finite positive measure and v be a positive measure on a measurable space (X, S). Prove that either $\mu \perp v$ or there exists a set $A \in S$ with $\mu(A) > 0$ and $c > 0$ such that A is a positive set for $v - c\mu$. 6

5. Let $(X\times Y, \sigma(S\times T), \mu\times v)$ be the product measure space of two σ-finite measure spaces (X, S, μ) and (Y, T, v). Let w be a $\mu\times v$-integrable, $\sigma(S\times T)$ measurable function on $X\times Y$. Prove that w^y is μ-integrable on X for v a.e. $y \in Y$ and

$$\int_{X\times Y} w d(\mu\times v) = \int_X \left[\int_Y w^y d\mu \right] dv. \qquad 6$$

6. Define positive, negative and null sets in a signed measure space (X, S, λ). If $\{A_n\}$ is an increasng sequence of positive sets for λ, prove that $\lim A_n$ is also a positive set for λ. 6

7. Verify the following statements about signed measures on a σ-algebra S of subsets of a set X :

(i) $\mu \perp w$ and $v \perp w \Rightarrow |\mu| + |v| \perp w$.

(ii) $\mu \perp w$ and $|v| \leq |\mu| \Rightarrow v \perp w$. 6

8. When is a signed measure space (X, S, λ) called finite ? Prove that $|\lambda|$ is finite on (X, S) if and only if λ is finite on (X, S). 6

Section - C

9. Let $\{A_n\}$ be a sequence of subsets of a set X. Prove that

$$\liminf A_n \subset \limsup A_n. \qquad 2$$

10. Given a signed measure space (X, S, λ). If f is a λ-semi-integrable function on X, show that

$$\left| \int_X f d\lambda \right| \leq \int_X |f| \, d|\lambda|. \qquad 2$$

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MATHEMATICS

PAPER - VI

Unit - II

Time - Two Hours | Full Marks - 40

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

Answer *one* from **Section - A**, *three* from **Section - B** and *all* from **Section - C**.

Section - A

1. (a) Show that any bounded linear functional on a subspace of a normed linear space can be extended continuously over the whole space without changing its norm. 12

(b) Show that for any $b \leq 0$ $\exists$ a bounded linear functional f s.t. $\| f \| = b$. 3

(c) Show that dual separates the points of the underlying normed linear space. 3

2. (a) Show that the linear space $B(X, Y)$ is a Banach space if Y is so. As a consequence of it show that dual of a normed linear space is always a Banach space. 7+2=9

(b) Show that every metric space has a unique completion. 9

Section - B

3. Show that the normed linear space $(c[0, 1], \|.\|)$ where $\|f\| = \int_0^1 |f(t)|\, dt \ \forall f \in c[0, 1]$ is not a Banach space. 6

4. Show that $T : DC[0, 1] \to C[0, 1]$ defined by $T(f) = \frac{df}{dx} \ \forall f \in DC[0, 1]$ is closed but not bounded, where $DC[0, 1]$ is the collection of all real valued functions on $[0, 1]$ where first derivative is continuous on $[0, 1]$. 6

5. Find the dual of l_p^n, $1 < p < \infty$. 6

6. Show that a metric space is complete if and only if intersection of nested sequence $\{F_n\}$ of nonempty closed sets with diam $(F_n) \to 0$ is singleton. 6

7. Let $\|.\|_1$ and $\|.\|_2$ be equivalent norms on a linear space X. Show that a sequence $\{x_n\}$ is Cauchy in $(X, \|.\|_1)$ if and only if it is so in $(X, \|.\|_2)$. 6

Section - C

8. Show that if in an inner product space $(X, \langle \cdot, \cdot \rangle)$, $\| x_n \| \to \| x \|$ and $\langle x_n, x \rangle \to \langle x, x \rangle$ then $x_n \to x$.

Also show that omission of any one condition creates a confusion regarding the truth of the result. 2+1+1=4

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MATHEMATICS

Paper - VII

Unit - I

Time : Two Hours Full Marks : 40

The figures in the margin indicate full marks.

Symbols have their usual meanings

Answer any one from Section A, three from Section B and all from section C.

Section - A

1. (a) Solve the one-dimensional wave equation satisfying given initial and boundary conditions by the method of separation of variables.

(b) The points of trisection of a string are pulled aside through a distance h on opposite sides of the position of equilibrium and the string is released from rest. Show that the middle point of the string remains always at rest. 10+8

2. Discuss the method of characteristics for solving the Cauchy problem for a first order quasi-linear equation. Find the integral surface of the equation

$2y(z - 3)p + (2x - z)q = y(2x - 3)$, passing through the circle $z = 0$, $x^2 + y^2 = 2x$. 10+8

Section - B 6×3=18

3. Show that the equations

$z = px + qy$ and $2xy(p^2 + q^2) = z(yp + xq)$ are compatible and hence find their common solution.

4. Verify integrability and hence solve

$z^2 dx + (z^2 - 2yz)\, dy + (2y^2 - yz - xz)\, dz = 0.$

5. Reduce into the canonical form :

$y(x+y)(z_{xx} \quad z_{yy}) - xz_x - yz_y - z = 0.$

6. Distinguish between Dirichlet and Neumann BVP for the Laplace equation. Show that the Neumann BVP is unique up to an additive constant.

7. Solve $u_t = u_{xx}$ satifying

$u(0, t) = u(l, t) \quad 0 \ \forall\, t > 0$

and $u(x. 0) = lx - x^2, 0 \leq x \leq l.$

8. Find the complete integral of

$p^2 + q^2 = 2.$

Hence find the integral surface passing through $x = 0$, $z = y$.

Section - C 2×2=4

9. Explain the meaning : well-posed problem.

10. Define Menge cone for a nonlinear first order p.d.e.

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MATHEMATICS

PAPER - VII

Unit - II

Time - Two Hours Full Marks - 40

The figures in the margin indicate full marks.

Symbols have their usual meanings.

Answer *one* from **Section - A**, *three* from **Section - B** and *all* from **Section - C**.

Section - A $18\times1=18$

1. State and explain D'Alembert's principle. Use this principle to derive Lagrange's equations of motion for a holonomic system. Hence find the equation of motion of a simple pendulum. 4+11+3=18

2. (a) Define cyclic coordinate. Show that the momentum conjugate to a cyclic coordinate is a constant of motion. 4

(b) Discuss the significance of Rauthian. Construct the Rauthian for the motion of a particle under a central force. 7+7=14

Section - B $6 \times 3 = 18$

3. Define canonical transformation. Show that

$$Q = q \cot p,\ P = \log\left(\sin \frac{p}{q}\right)$$

is a canonical transformation.

4. A particle of mass m is projected with initial velocity u at an angle α with the horizontal axis. Use Lagrange's equations to describe the motion. The air resistance may be neglected.

5. State Hamilton's principle. Hence deduce Hamilton's equations of motion.

6. Find the attraction of a uniform thin rectangular plate upon a unit mass situated on the perpendicular line to the plate through the centre.

7. Show that a catenary revolved about its axis yields the minimum surface of revolution.

8. State and prove the Gauss theorem on the flux of force across a closed surface.

Section - C $2 \times 2 = 4$

9. Show that $[q_i, p_j] = \delta_{ij}$, $[t, H] = 1$.

10. Find the gravitational potential of a system of n particles.

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MATHEMATICS

Paper - VIII

Unit - I

Time : Two Hours | Full Marks : 40

The figures in the margin indicate full marks.

Symbols have their usual meanings

Answer any one from Section A, three from Section B and all from Section C.

Section - A

1. (a) Derive the expression of complex potential due to a source at the origin. 5

(b) Derive the expression of complex potential for a doublet of strength μ at the origin directed along the x-axis. 5

(c) Determine the kinetic energy when an elliptic cylinder rotates in an infinite mass of liquid at rest at infinity. 8

2. (a) In an incompressible fluid the vorticity at every point is constant in magnitude and direction. Prove that the componenis of velocity u, v, w are the solutions of Laplace's equation. 6

(b) Find the complex potential if an infinite row of parallel straight vortices, each of strength k, be situated at the points $O, \pm a, \pm 2a, \ldots, \pm na, \ldots$.

Also, find the velocity components at any point of the fluid. 7

(c) Find the necessary and sufficient condition that vortex lines may be at right angles to the streamlines. 5

Section - B

3. Derive the equation of continuity in cartesian coordinates. 6

4. Show that the velocity potential $\phi = \frac{1}{2} a (x^2 + y^2 - 2z^2)$ satisfies the Laplace equation. Also, determine the streamlines. 6

5. Show that in the motion of a fluid in two dimensions if the coordinates (x, y) of an element at any time be expressed in terms of the initial coordinates (a, b) and the time, the motion is irrotational if,

$$\frac{\partial(\dot{x}, x)}{\partial(a,b)} + \frac{\partial(\dot{y}, y)}{\partial(a,b)} = 0 .$$ 6

6. Determine the motion of a circular cylinder in an infinite mass of the liquid at rest at infinity with velocity U parallel to the x-axis. 6

7. If $udx + vdy + wdz = d\theta + \lambda d\mu$, where λ, θ, μ are functions of x, y, z and t, prove that the vortex lines at any time are the lines of intersection of the surfaces λ= constant and μ = constant. 6

8. Show that the general motion of a fluid element is made up of pure translation, pure deformation and pure rotation. 6

Section - C

9. (a) Define vortex lines. 2

(b) What arrangements of sources and sinks will give rise to the function

$$W = \log\left(z - \frac{a^2}{z}\right) ?$$ 2

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MATHEMATICS

Paper - VIII

Unit - II

Time : Two Hours Full Marks : 40

The figures in the margin indicate full marks.

Symbols have their usual meanings

Answer any one from Section A, three from Section B and all from section C.

Section - A

1. (a) Define principal strains. Show that the first strain invariant represents cubical dilatation. 2+6

(b) Obtain the principal stresses and principal stress directions if the stress tensor at a point is given by

$$\begin{pmatrix} 5 & 2 & 2 \\ 2 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$ 6

(c) Prove that the principal axes of stress are coincident with the principal axes of strain when the medium is isotropic. 4

2. (a) Derive von Karman momentum integral equation for two-dimensional incompressible boundary layers. 12

(b) Discuss the steady flow in pipes of equilateral triangular section. 6

Section - B

3. For a velocity distribution in a boundary layer

$$\frac{u}{U} = \alpha + \beta\,(y/\delta) + \gamma\,(y/\delta)^2 + \varepsilon\,(y/\delta)^3,$$

determine the constants α, β, γ, ε. Also, find the displacement thickness and skin friction coefficient. 6

4. Prove that the two principal directions corresponding to any two distinct principal stresses are orthogonal. 6

5. Discuss the pulsatile flow between two parallel surfaces which are situated at a distance $y = \pm a$ apart and the pressure gradient be in the X - direction to oscillate in time t. 6

6. Derive energy-integral equation for two-dimensional laminar boundary layers in incompressible flows. 6

7. For the flow of a fluid through similar pipes, show that

$$P = (\rho l v^2/d)\, \phi\, (v d \rho/\mu),$$

where P is the drop in pressure over the length l of the pipe, d is the diameter of the pipe, ρ is the density, μ is the viscosity of the fluid and v is the mean velocity of the flow through the pipe. 6

8. Give the physical significance of the following non-dimensional parameters :

(i) Péclet number, (ii) Mach number, (iii) Grashof number, (iv) Euler's number. 4×1.5

Section - C

9. (a) Explain the principle of dynamical similarity.

(b) Define momentum thickness. 2

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MATHEMATICS

Paper - IX

Unit - I

Time : Two Hours Full Marks : 40

The figures in the margin indicate full marks.

Section - A

1. (a) What is the difference between 'ordinary derivative' and 'covariant derivative' of a vector ? Illustrate.

(b) What is a tensor ? Write quotient law. 2+2

Section - B

Answer *any three* questions.

2. (a) State the principle of equivalence.

(b) Write down Bianchi identity. Using Bianchi identity obtain Einstein's field equation for matter free condition. 2+4

3. (a) What is time dilation ?

(b) Describe Pound-Rebka experiment to verify gravitational redshift formula. 2+4

4. Obtain the condition for existence of circular orbits for motion in Schwarzschild metric. 6

5. Starting from the geodesic principle, show that the equation of motion for a test particle (of non-zero rest mass) in a gravitational field is given by

$$\frac{d^2 x^{\lambda}}{ds^2} + \Gamma^{\lambda}_{\mu\nu} \frac{dx^{\mu}}{ds} \cdot \frac{dx^{\nu}}{ds} = 0,$$

where ds is the arc length of the space-time. What is the trajectory of a particle in flat space ? Deduce using the geodesic equation. 4+2

6. Obtain Lane-Emden equation starting from the hydrostatic equilibrium of a Newtonian star with polytropic equation of state. 6

7. What is Shapin effect ? Determine the total time delay of a rader signal for the travel to Mercury and back to earth. 1+5

Section - C

Answer *any one* question.

8. (a) Find the integrals of motion for a test particle in Schwarzschild space-time. Show that general relativity predicts precession of planetary orbits and hence obtain the precession frequency.

(b) Show that the total deflection of light rays in a gravitational field is

$$\Delta\phi = \frac{4M}{R}$$

with $G = 1$ and $c = 1$. Hence determine the deflection angle for a ray just grazing the sun. (6+6)+6

9. (a) What is a Newtonian star ? Show that the total energy of a Newtonian star is

$$E = -\frac{3\gamma - 4}{5\gamma - 6}\,\frac{M^2}{R}$$

where $\gamma = C_p / C_v$.

(b) Determine the space-time metric outside a charged and spherically symmetric geometry. 6+12

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MATHEMATICS

PAPER - IX

Unit - II

Time - Two Hours Full Marks - 40

The terms have their usual meanings.

Section - A

Answer *all* questions. 2×2=4

1. From the baryon conservation law in the Friedmann universe, deduce that
$T(t)\, R(t) = \text{constant}.$ 2

2. Give the arguments of Brans and Dicke leading to $G \sim \phi^{-1}$. 2

Section - B

Answer *any three* questions. 3×6=18

3. Calculate the luminosity distance D_l in the closed model of the Friedmann universe ($k = +1$).

4. Deduce the cosmological redshift formula

$$1 + z = \frac{R(t_0)}{R(t_1)}$$

5. Show that for $k = +1$, the universe is a 3-sphere with radius $R/\sqrt{k}$ imbedded in a fictitious 4-dimensional Euclidean space. Also show that the 3-volume of this universe is

$$V = \frac{2\pi^2 R^3}{k^{3/2}}.$$

6. Show that the deceleration parameter q is related to the observed present density ρ_0 and the critical density ρ_c by the relation

$$2q = \frac{\rho_0}{\rho_c}.$$

7. Find different scale factors by solving the dynamical equations for the non-static Friedman-Robertson-Walker models for $k = 0$, $\Lambda = 0$ using the equation of state

$$p_0 = (\gamma - 1)\,\rho_0 c^2,\ 1 \le \gamma \le 2.$$

8. Describe Olber's paradox and state briefly how it is resolved by Friedmann cosmology.

Section - C

Answer *any one* question. 1×18=18

9. State Cosmological Principle. Using it, briefly sketch the derivation of the Robertson-Walker metric.

10. Deduce that the apparent bolometric luminosity of a galaxy with radial comoving coordinte r_1 is given by

$$\mathcal{F}_{bol} = \frac{L_{bol}}{4\pi D^2 (1+z)^2},$$

where $D_1 = r_1 R(t_0)$ is the luminosity distance.

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MATHEMATICS

PAPER - X

Unit - I

Time - Two Hours　　　　　　　　Full Marks - 40

The figures in the margin indicate full marks.

Section - A

1. State Balmer's law. 2

2. State Planck's law of radiation. 2

Section - B

Answer *any three* questions. $6\times3=18$

3. Write a short note on hydrogen spectra.

4. Discuss Eddington second approximation of the equation of radiation transfer.

5 Deduce the expression for the absorption coefficient when the lines are broadened.

6. Deduce the equation of radiation transfer for coherent scattering.

7. Discuss the influence of Doppler's effect on absorption coefficient.

8. Write a note on non-coherent scattering.

Section - C

Answer *any one* question. 1×18=18

9. Solve the equation of radiation transfer by Eddington method for continuous spectrum.

10. Deduce the expression

$$I(\gamma) = \frac{1}{\pi} \frac{\gamma/4\pi}{(\gamma - \gamma_0)^2 + (\gamma/4\pi)^2}.$$

[All symbols have their usual meanings]

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Paper - X

Unit - II

Time : Two Hours Full Marks : 40

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Symbols have their usual meanings

Answer any one from Section A any three from Section B and all from Section C.

Section - A 18×1=18

1. Solve the following equation of transfer

$$\mu \frac{dI}{d\tau}(\tau\mu) = I(\tau, \mu) - B(\tau)$$

by Ambertzumian method and find expression for $I(0, \mu)$.

2. Establish the following relation :

$$H(\mu)\, H(-\mu) = \frac{1}{T(\mu)};$$

using the limits of solutions given by the method of discrete ordinates as $n \to \infty$.

Section - B $3\times6=18$

1. Prove that $\log H_n(\mu) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \frac{\mu \log T_n(w)}{w^2 - \mu^2}\,dw$.

2. Show that

$$X(\mu)T(\mu) = 1 - \mu \int_0^1 \frac{\psi(u)T(u)}{\mu - \mu}\,du - \mu \exp\left(-\frac{\tau_1}{\mu}\right)$$

$$\int_0^1 \frac{\psi(u)Y(u)}{\mu + \mu}\,du.$$

3. Show that $1 - x_0 = \left\{1 - 2\psi_1 + y_0^2\right\}^{\frac{1}{2}}$.

4. Establish $K_1K_2 \ldots K_{n-1}M_1M_2 \ldots M_n = \frac{1}{\sqrt{3}}$.

5. Prove that $\lim_{\mu \to \infty} Y(\mu) = \lim_{\mu \to \infty} X(\mu) = J(0, 0)$.

6. Find the two first integrals for SPM for anisotropic equation of transfer.

Section - C $2\times2=4$

1. (a) Write a short note on L. T. E.

   (b) Write a short note on $B(\tau)$.
