# M. Sc. Part. II
# Mathematics Syllabus

| | | | Full Marks |
|---|---|---|---|
| **Paper VI:** | **Unit I :** Integration Theory | : | 40 |
| | **Unit II :** Functional Analysis | : | 40 |
| **Paper VII:** | **Unit I :** Partial Differential Equations | : | 40 |
| | **Unit II :** Mechanics | : | 40 |
| **Paper VIII:**. | **Unit I :** Fluid Mechanics I | : | 40 |
| | **Unit II :** Fluid Mechanics II | : | 40 |
| **Paper IX:** | **Unit I :** General Relativity I | : | 40 |
| | **Unit II :** Cosmology II | : | 40 |
| **Paper X:** | **Unit I :** Theoretical Astrophysics I | : | 40 |
| | **Unit II :** Theoretical Astrophysics II | : | 40 |
| | **Assignment** | : | 100 |

## PAPER VI

## UNIT I : INTEGRATION THEORY

Signed measure. Hahn decomposition theorem, mutually singular measures. Radon-Nikodym theorem. Lebesgue decomposition. Riesz representation theorem. Extension theorem (Caratheodory). Lebesgue-Stieltjes integral, product measures. Fubini's theorem. Differentiation and Integration. Decomposition into absolutely continuous and singular parts.

Baire sets. Baire measure, continuous functions with compact support. Regularity of measures on locally compact spaces. Integration of continuous functions with compact support. Riesz-Markoff theorem.





## UNIT II : FUNCTIONAL ANALYSIS

Normed linear spaces. Banach spaces and examples. Quotient space of normed linear spaces and its completeness, equivalent norms. Riesz Lemma. Basic properties of finite dimensional normed linear spaces and compactness. Weak convergence and founded linear transformation normed liner spaces of bounded linear transformations, dual spaces with examples. Uniform boundedness theorem and some of its consequences. Open mapping and closed graph theorems. Hahn-Banach theorem for real linear spaces. Complex linear spaces and normed linear spaces. Reflexive spaces. Weak Sequential compactness. Compact Operators. Solvability of linear equations in Banach spaces. The closed Range Theorem.

Inner product spaces. Hilbert spaces. Orthonormal sets. Bessel's inequality. Complete orthonormal sets and Parseval's identity. Structure of Hilbert spaces. Projection theorem. Riesz representation theorem. Adjoint of an operator on a Hilbert space, Reflexivity of Hilbert spaces. Self-adjoint operators. Positive projection normal and unitary operators. Abstract variational boundary-value problem. The generalized Lax-Milgram theorem.

**References:**


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21. R.B. Holmes. Geometric Functional Analysis and its Applications. Springer-verlag 1975
22. K. K. Jha. Functional Analysis Students Friends 1986
23. L.V. Kantorovich and G. P. Akilov. Functional Analysis. Pergamon Press 1982
24. K. Kreyszig. Introductory Functional Analysis with Applications. John Wiley & Sons New York. 1978
25. B. K. Lahiri. Elements of Functional Analysis. The World Press Pvt. Ltd. Calcutta. 1994.
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34. A Wilansky. Functional Analysis. Blaisdell Publishing Co., 1964.
35. J. Tinsley Oden & Leszek F. Demkowicz. Applied Functional Analsysis CRC Press Inc 1996.
36. A. H. Siddiqui. Functional Analysis with Applicatios. Tata McGra Hill Pub. Co. New Delhi.

## PAPER VII

## UNIT I : PARTIAL DIFFERENTIAL EQUATIONS

**Examples of PDE Classification**

Transport Equation – Initial value problem. Non-homogeneous equations. Laplace's Equation-Fundamental Solution. Mean Value formulas. Properties of Harmonic Functions, Green's Function. Energy Melthods.

Heat Equation – Fundamental Solution. Mean Value Formula. Properties of Solutions. Energy Methods.

Wave Equation-Solution by Spherical Means. Non-homogeneous Equations. Energy methods. Nonlinear First Order PDE-Complete Integrals. Envelopes. Characteristics. Hamilton-Jacobi Equations (Calculus of Variations. Hamilton's ODE. Legendre Transform. Hopf-LaxFormula. Weak Solutions. Uniqueness). Conservation Laws (Shocks. Entropy Condition. Lax-Oleinik formula. Weak Solutions. Uniqueness. Riemann's Problem. Long Time Behaviour).

Representation of Solutions-Separation of Variables. Similarity Solutions (Plane and Travelling Waves. Solitons. Similarity under Scaling). Fourier and Laplace Transform. Hopf-Cole Transform. Hodograph and Legendre Transforms. Potential Functions. Asymptotics (singular Perturbations. Laplace's Method. Geometric Optics. Stationary Phase. Homogenization) Power Series (Non-Characteristic Surfaces. Real Analytic Functions. Cauchy-Kovalevskaya Theorem).

## UNIT II : MECHANICS

### Analytical dynamics

Generalized coordinates. Holonomic and Non-holonomic systems. Scleronomic and Rheonomic systems. Generalized potential. Lagramge's equations of first kind. Lagrange's equations of second kind. Uniqueness of solution. Energy equation for conservative fields.

Hamilton's variables. Donkin's theorem. Hamilton canonical equations. Cyclic coordinates. Routh's equations. Poisson's Bracket. Poisson's Identity. Jacobi-Poisson Theorem. Motivating problems of calculus of variations. Shortest distance. Minimum surface of revolution. Brachistochrone problem. Isoperimetric problem. Geodesic. Fundamental lemma of calculus of variations. Eluler's equation for one dependent function and its generalization to (i) in dependent functions. (ii) higher order derivatives. Conditional extrememum under geometic constraints and under integral constraints.

Hamilton's Principle. Principle of least action. Poincare Cartan Integral invariant. Whittaker's equations. Jacobi's equations. Statement of Lee Hwa Chung's theorem.

Hamilton-Jacobi equation Jacobi theorem. Method of separation of variables. Lagrange Brackets. Condition of canonical character of a transformation in terms of Lagrange brackets and Poisson brackets. Invariance of Lagrange brackets and Poisson brackets under canonical transformations.

### Gravitation

Attraction and potential of rod disc. spherical shells and sphere. Surface integral of normal attractions (application & Gauss theorem). Laplace and Poisson equation. Work done by self attracting systems. Distributions for a given potential. Equipotential surface. Surfaces and solid harmonics. Surface density in terms of surface harmonics.

### References

1. L.C. Evans. Partial Differential Equations. Graduate Studies in Mathematics. Volume 19, AMS. 1998.
2. Books with the above title by I. N. Sneddon. F. John. P. Prasad an R. Ravindran. Amarnath etc.
3. A.S. Ramsey. Dynamics Part II. The English Language Book Society and Cambridge Univ. Press. 1972.
4. F. Gantmacher. Lectures in Analytic Mechanics. MIR Publishers. Moscow. 1975
5. H. Goldstein. Classical Mechanics (2nd edition). Narosa Publishing House. New Delhi.
6. I.M. Gellfand and S.V. Fomin. Calculus of Variations. Prentice Hall.
7. S.L. Lonley. An Elementary Treatise on Statics. Kalyani Publishers. New Delhi 1979.
8. A.S. Ramsey, Newtonian Gravitation. The Eglish Language Book Society and Cambridge Univ. Press.
9. Narayan Chandra Rana & Pramod Sharad Chandra Joag. Classical Mechanics. Tata McGraw Hill. 1991.
10. 10. Louis N. Hand and Janet D. Finch. Analytical Mechanics. Cambridge University Press, 1997.





## PAPER VIII

## UNIT I : FLUID MECHANICS I

Kinematics-Lagrangian and Eulerian methods. Equation of continuity. Boundary surfaces. Stream lines. Path lines and streak lines Velocity potential. Irrotational and rotational motions. Vortex lines.

Equations of Motion-Lagrange's and Euler's equations of motion. Bernoulli's theorem. Equation of Motion by flux method. Equations referred to moving axes. Impulsive actions. Stream function. Irrotational motion two-dimensions. Complex velocity potential. Sources, sinks, doublets and their images. Conformal mapping. Milne-Thomson circle theorem.

Two-dimensional irrotational motion in produced by motion of circular coaxial and elliptic cylinders in an finite mass of liquid, Kinetic energy of liquid. Theorem of Blasius motion of a sphere through a liquid at rest at infinity. Liquid streaming past a fixed sphere. Equation of motion of a sphere. Stoke's stream function. Vortex motion and its elementary properties Kelvin's proof of permanence. Motion due to circular and rectilinear vortices.

## UNIT II : FLUID MECHANICS II

Wave motion in a gas. Speed of Sound Equation of motion of a gas. Subsonic. sonic and supersonic flows of a gas. Isentropic gas flows. Flow through a nozzle. Normal and oblique shocks.

Stress components in a real fluid. Relations between rectangular components of stress. Connection between stresses and gradients of velocity. Navier-stoke's equations of motion. Plane Poiseuille and Couette flows between two parallel plates. Theory of Lubrication. Flow through tubes of uniform cross section in from of circle, annulus, ellipse and equilateral triangle under constant pressure gradient. Unsteady flow over a flat plate.

Dynamical similarity. Buckingham p-theorem. Reynolds number. Prandt's boundary layer. Boundary layer equations in two-dimensions. Blasius solution. Boundary layer thickness. Displacement thickness. Karman integral conditions. Separation of boundary layer flow.

**References:**


1. W.H. Besaint and A.S. Ramsey. A. Treatise on : Hydromechanics. Part-II. CBS Publishers Delhi-1988.
2. G.K. Batchelor, An Introduction to Fluid Mechanics. Foundations Books, New Delhi, 1994.
3. F. Chorlton. Textbook of Fluid Dynamics. CBS Publishers. Delhi – 1985
4. A. J. Chorin and A. Marsden. A. Mathematical Introduction to Fluid Dynamics Springer-Verlag. New York-1993.
5. L.D. Landau and E. M. Lipschitz. Fluid Mechanics Pergamon Press. London. 1985.
6. H. Schilichting Boundary Layer Theory. McGraw Hill Book Company, New York, 1979
7. R. K. Rathy, An Introduction to Fluid Dynamcis. Oxford and IBH Publishing Company, New Delhi, 1976
8. A.D. Young Boundary Layers. AIAA Education Series. Washington DC. 1989.
9. S.W. Yuan. Foundations of Fluid Mechanics. Prentice Hall of India Private Ltd. New Delhi 1976.






## PAPER - IX

## UNIT I : GENERAL RELATIVITY

General Relativity – Transformation of coordinates. Tensors. Algebra of Tensors. Symmetric and skew symmetric Tensors. Contraction of tensors and quotient law.

Riemannian metric. Parallel transport. Christoffel Symbols. Covarient derivatives. Intrinsic derivatives and geodesics. Riemann Christoffel curvature tensor and its symmetry properties. Bianchi identities and Einstein tensor.

Review of the special theory of relativity and the Newtonian Theory of gravitation Principle of equivalence and general covariance geodesic principle. Newtonian approximation of relativistic equations of motion. Einstein's field equations and its Newtonian approximation.

Schwarzschild external solution and its isotropic form. Planetary orbits and anologues of Kepler's Laws in general relativity. Advance of perihelion of a planet. Bending of light rays in a gravitational field. Gravitational redshift of spectral lines. Radar echo delay.

Energy-momentum tensor of a perfect fluid. Schwarzschild internal solution. Boundary conditions. Energy momentum tensor of an electromagnetic field. Einstein-Maxwell equations. Reissner-Nordstrom solution.

## UNIT II : COSMOLOGY

Cosmology-Mach's principle. Linstein modified field equations with cosmological term. Static Cosmological models of Einstein and De-Sitter, their derivation properties and comparison with the actual universe.

Hubble's law Cosmological principles. Weyl's postulate. Derivation of Robertson-Walker metric Hubble and deceleration parameters. Redshift. Redshift versus distance relation. Angular size versus redshift relation and source counts in Robertson-Walker space-time.

Friedmann models. Fundamental equations of dynamical cosmology. Critical density. Closed and open Universe. Age of the universe. Matter dominated era of the Universe. Einstein-de-sitter model Particle and even horizons.

Eddington-Lamaitre models with I-term. Perfect cosmological principle. Steady state cosmology.

**Refernces:**


1. C.E. Weatherburn. An Introduction to Riemannian Geometry and the tensor Calculus. Cambridge University Press. 1950.
2. H. Stepheni General Relativity: An Introduction to the theory of the gravitational field. Cambridge University Press. 1982.
3. A.S. Eddington. The Mathematical Theory of Relativity. Cambridge University Press. 1965.
4. J.V. Narlikar. General Relativity and Cosmology: The Macmillon Company of India Limited 1978.
5. R. Adlev. M. Bazin. M. Schiffer. Introduction to general relativity. McGraw Hill Inc. 1975.
6. B. F. Shutz. A first course 19n general relativity. Cambridge University Press. 1990.
7. S. Weinberg. Gravitation and Cosmology; Principles and applications of the general theory of relativity, John Willey & Sons. Inc. 1972.
8. J. V. Narlikar. Introduction to Cosmology. Cambridge University Press. 1993.
9. R.K. Sachs and H.Wu. General Relativity for Mathematician. Springer Verlag. 1977.
10. L.D. Landau and E.M. Lifshitz. The classical theory of Fields. Pergamon Press, 1980.
11. J. L. Synge. Relativity: The general theory. North Holland Publishing Company, 1976.


## PAPER - X

### UNIT I : THEORETICAL ASTROPHYSICS I

Absorption lines. Hydrogen spectrum. Specific intensity, absorption, emission, optical depth. Radiative equilibrium local thermodynamic equilibrium, phase function, scattering equation of transfer, solution of equation of transfer by Eddington's method. Physical phenomena in the atmosphere-classical oscillator, absorption coefficient emission effect on absorption lines, quantum atom. Coherent scattering, equation of transfer for non-coherent scattering, interlocking of lines. Equation of transfer for non-coherent scattering and interlocked multiplates and their solutions.





### UNIT II : THEORETICAL ASTROPHYSICS II

Methods of solution – Chandrasekhar's discrete ordinate method Chandrasekhar's principle of invariance. Milne's method. Ambertzumian method, spherical harmonic method, modified spherical harmonic method. Application of the methods in solving transport equation. H-function. X-andY-function. Voigt function.

**References:**


1. Woolley & Stibbs. Outer Layers of a Star
2. Chandrashekhar. S., Radiative Transfer
3. Busbridge, I. W. Mathematics of Radiative Transfer
4. Kourganoff. V., Basic Methods Transfer Problems.
5. White, Atomic Spectra


**Assignment 100 Marks**

***End***